



TITLE:

# Numerical study of the second harmonic generation of Lamb waves at an imperfect joint of plates

AUTHOR(S):

Mori, Naoki; Biwa, Shiro; Kusaka, Takayuki

---

CITATION:

Mori, Naoki ...[et al]. Numerical study of the second harmonic generation of Lamb waves at an imperfect joint of plates. Proceedings of Meetings on Acoustics 2018, 34(1): 030002.

ISSUE DATE:

2018-07-09

URL:

<http://hdl.handle.net/2433/243196>

RIGHT:

© 2018 Acoustical Society of America; The full-text file will be made open to the public on 09 July 2019 in accordance with publisher's 'Terms and Conditions for Self-Archiving'.



# Proceedings of Meetings on Acoustics

Volume 34

<http://acousticalsociety.org/>

## 21<sup>st</sup> International Symposium on Nonlinear Acoustics

**21<sup>st</sup> ISNA**

Santa Fe, New Mexico, USA

July 9-13, 2018, 50<sup>th</sup> Anniversary

### Engineering Acoustics: Paper S13-1

## Numerical study of the second harmonic generation of Lamb waves at an imperfect joint of plates

**Naoki Mori***Department of Mechanical Engineering, College of Science and Engineering, Ritsumeikan University, Kusatsu, Shiga, 5258577, JAPAN; nmori@fc.ritsumei.ac.jp***Shiro Biwa***Department of Aeronautics and Astronautics, Graduate School of Engineering, Kyoto University, Kyoto, 6158540, JAPAN; biwa@kuaero.kyoto-u.ac.jp***Takayuki Kusaka***Department of Mechanical Engineering, College of Science and Engineering, Ritsumeikan University, Kusatsu, Shiga, 5258577, JAPAN; kusaka@se.ritsumei.ac.jp*

Lamb waves are attracting significant attention in the nondestructive evaluation for plate structures due to their capability to propagate long distances. In this study, the nonlinear interaction of Lamb waves with an imperfect joint of elastic plates is numerically investigated. In particular, the second harmonic generation behavior from the joint is examined by perturbation analysis. The imperfect joint is modeled as a spring-type interface with quadratic nonlinearity, and the perturbation analysis is carried out using the hybrid finite element method (HFEM). For the incidence of the lowest-order symmetric (S0) Lamb mode below the cut-off frequencies of the higher-order modes, the double-frequency S0 mode is generated from the imperfect joint due to the nonlinear interaction. A nonlinear parameter calculated from the amplitude of the second harmonic S0 mode shows a sharp peak at a certain incident frequency. The peak frequency increases with increasing interfacial stiffness, and the double of the peak frequency corresponds to the resonance frequency of the imperfect joint subjected to the S0 mode incidence. This result shows that the resonance of the imperfect joint leads to the amplification of the second harmonic S0 mode.

Published by the Acoustical Society of America



## 1. INTRODUCTION

Ultrasonic waves are widely utilized in the area of the nondestructive evaluation and the health monitoring of various structures. When a crack-type defect is subjected to compressive loading, the partial contact of the crack surfaces often occurs and gives rise to difficulty in detecting the defect by the conventional ultrasonic method based on the reflection and transmission characteristics. To enhance the sensitivity to closed defects, nonlinear features of ultrasonic waves interacting with a contacting interface were extensively explored in previous studies.<sup>1, 2</sup>

Lamb waves, which are guided waves in plates, provide an effective means of the nondestructive evaluation for plate structures because they can propagate relatively long distances. The reflection and transmission behavior of the Lamb wave at a butt-type imperfect joint of plates, such as an adhesive joint and a contacting interface, was investigated theoretically and experimentally in foregoing studies.<sup>3-6</sup> The detection of a closed crack based on the nonlinear behavior of the Lamb wave, such as higher harmonic generation from the crack surfaces, was carried out in previous studies.<sup>7, 8</sup> However, fundamental aspects of the nonlinear interaction of Lamb waves with an imperfect joint have not been sufficiently revealed yet. The objective of this paper is to examine the second harmonic generation behavior from an imperfect joint of elastic plates subjected to the Lamb wave incidence. To this purpose, the perturbation analysis<sup>9</sup> is performed using the hybrid finite element method (HFEM).<sup>3</sup>

## 2. PERTURBATION ANALYSIS

### A. NUMERICAL MODEL

As shown in Fig. 1, linear elastic and homogeneous plates of thickness  $d$  are imperfectly jointed at  $x_1 = 0$  in the  $x_1$ - $x_2$  coordinates. The plates are under the plane-strain condition in the  $x_1$ - $x_2$  plane, and the imperfect joint is subjected to the incidence of the Lamb wave propagating in the  $x_1$  direction. In this paper, the lowest-order symmetric (S0) mode is used as the incident wave. The propagation behavior of the Lamb wave is governed by two-dimensional Navier's equation

$$(c_L^2 - c_T^2) \frac{\partial}{\partial x_\alpha} \left( \frac{\partial u_\gamma}{\partial x_\gamma} \right) + c_T^2 \frac{\partial}{\partial x_\gamma} \left( \frac{\partial u_\alpha}{\partial x_\gamma} \right) - \frac{\partial^2 u_\alpha}{\partial t^2} = 0, \quad (1)$$

where  $t$  is time,  $u_\alpha(x_1, x_2, t)$  is the displacement component ( $\alpha = 1, 2$ ), and  $c_L$  and  $c_T$  are the velocities of the longitudinal and transverse waves, respectively. Two-dimensional summation convention is applied to the repeated Greek index  $\gamma$  in Eq. (1). On the plate surfaces  $|x_2| = d/2$ , the traction-free boundary condition is applied, i.e.  $\sigma_{\alpha 2} = 0$ , where  $\sigma_{\alpha\delta}(x_1, x_2, t)$  is the stress component ( $\alpha, \delta = 1, 2$ ). The imperfect joint  $x_1 = 0$  is modeled as a spring-type interface with quadratic nonlinearity, which yields the interfacial stress components using the displacement discontinuity  $[u_\alpha] = u_\alpha(0^+, x_2, t) - u_\alpha(0^-, x_2, t)$  at the interface as

$$\sigma_{11}(0^\pm, x_2, t) = K_N(1 - \beta[u_1])[u_1], \sigma_{12}(0^\pm, x_2, t) = K_T[u_2], \quad (2)$$

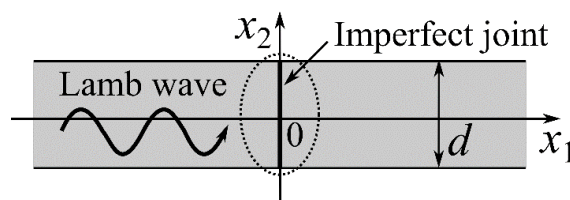


Figure 1. Schematic of the imperfect joint of elastic plates subjected to the Lamb wave incidence.

where  $K_N$  and  $K_T$  are normal and tangential stiffnesses, respectively, and  $\beta$  is the quadratic coefficient. In this study, only the quadratic term with respect to  $[u_1]$  is considered because the in-plane displacement of the S0 mode is dominant compared to the out-of-plane displacement. The interfacial parameters are assumed to be uniform in the thickness ( $x_2$ ) direction.

In the frequency domain, Eq. (1) is solved under the assumption of sufficiently weak nonlinearity  $\beta A_0 \ll 1$ , where  $A_0$  is the displacement amplitude of the incident wave. The displacement field is expressed as the combination of the fundamental wave and the second harmonic<sup>9</sup>

$$u_\alpha = u_\alpha^F + u_\alpha^S, \quad (3)$$

where  $u_\alpha^F$  and  $u_\alpha^S$  denote the displacements of the fundamental wave and the second harmonic, respectively. The displacement field of the fundamental wave  $u_\alpha^F$  is obtained from Eq. (1) for angular frequency  $\omega_0 = 2\pi f_0$  with a linearized boundary condition of the imperfect joint ( $\beta = 0$ ). The solution of the second harmonic  $u_\alpha^S$  is derived from Eq. (1) for double angular frequency  $2\omega_0$  with a boundary condition

$$\sigma_{11}^S = K_N[u_1^S] - \beta K_N[u_1^F]^2, \sigma_{12}^S = K_T[u_2^S], \quad (4)$$

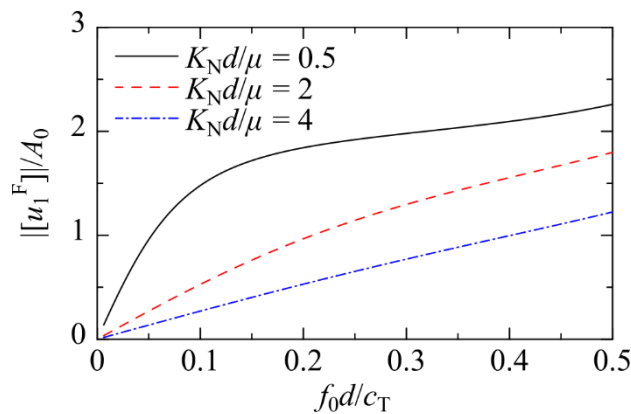
at  $x_1 = 0$ , where  $[u_1^F]$  denotes the interfacial displacement gap induced by the fundamental wave. For each angular frequency  $\omega_0 = 2\pi f_0$ , the displacement fields of the fundamental wave and the second harmonic are calculated in the above procedure. A low frequency range  $f_0 d/c_T < 0.5$  in which only the lowest-order symmetric (S0) and antisymmetric (A0) Lamb modes can propagate is considered in this study.

## B. HYBRID FINITE ELEMENT METHOD (HFEM)

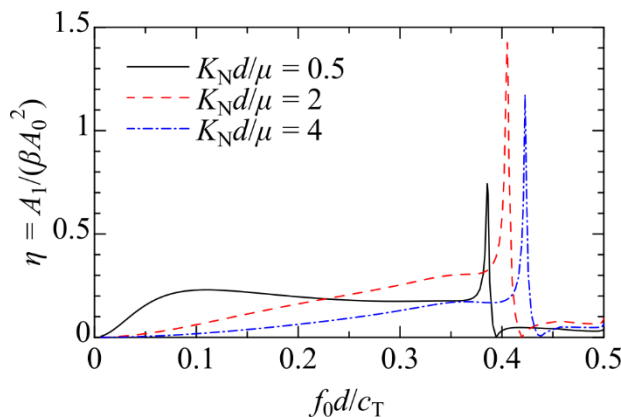
The interaction of the Lamb wave with the imperfect joint is analyzed by hybrid finite element method (HFEM).<sup>3</sup> In the formulation of the HFEM, a bounded region  $|x_1| < L/2$  is discretized by finite elements, while the displacement fields in the semi-infinite regions  $|x_1| > L/2$  are expressed as the superposition of different Lamb modes. The continuity condition of the displacement and stress components is applied at  $|x_1| = L/2$  and the amplitudes of the reflected and transmitted Lamb modes are calculated. In the present study, square-shaped four-node isoparametric elements with side length  $d/20$  were used for the discretization of the region  $|x_1| < L/2$ . The length of the bounded area  $L$  was set as  $L = d/2$ , and six symmetric and six antisymmetric Lamb modes were considered for the modal expansion in  $|x_1| > L/2$ . The total number of the nodes and the elements were 252 and 200, respectively. The material property of aluminum alloy ( $c_L = 6.40$  km/s,  $c_T = 3.17$  km/s, and  $\rho = 2.70 \times 10^3$  kg/m<sup>3</sup>) was used in the numerical analysis, where  $\rho$  denotes the mass density. The stiffness ratio  $K_T/K_N$  was fixed as  $K_T/K_N = 0.3$  for simplicity, and the product of the quadratic coefficient  $\beta$  and the incident displacement amplitude  $A_0$  was set as  $\beta A_0 = 0.01$  in this analysis.

## 3. NUMERICAL RESULTS

The perturbation analysis using the HFEM was performed for three normal stiffnesses  $K_N d/\mu = 0.5, 2$ , and 4, where  $\mu = \rho c_T^2$  is the shear modulus of the plate. The amplitude of the normal displacement gap at the interface induced by the fundamental wave  $[u_1^F]$  is shown as a function of the normalized incident frequency  $f_0 d/c_T$  in Fig. 2. The displacement gap is normalized by the incident displacement amplitude  $A_0$ . For each normal stiffness, the displacement gap increases monotonically with increasing incident frequency. Based on these results, the amplitude of the second harmonic S0 mode transmitted across the imperfect joint  $A_1$  is obtained from the in-plane displacement  $u_1$  on the middle surface of the plate  $x_2 = 0$ . In the perturbation analysis, the second harmonic amplitude  $A_1$  is proportional to the square of the incident amplitude  $A_0^2$ . Thus a nonlinear parameter  $\eta = A_1/(\beta A_0^2)$  is calculated from the second harmonic amplitude at each incident frequency, as shown in Fig. 3. In a sufficiently low frequency range, the nonlinear parameter  $\eta$  increases monotonically with increasing incident frequency. However, a sharp peak appears at a certain incident frequency around  $f_0 d/c_T = 0.4$ , which varies with the normal stiffness  $K_N d/\mu$ . This peak frequency  $F_n$  tends to increase as the normal stiffness increases.



**Figure 2.** Variation of the interfacial displacement gap amplitude by the fundamental wave with the normalized incident frequency, for different normal stiffnesses.

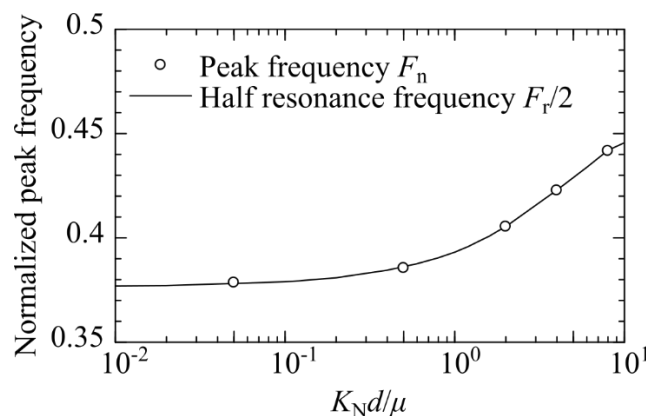


**Figure 3.** Variation of the nonlinear parameter calculated from the second harmonic amplitude with the normalized incident frequency, for different normal stiffnesses.

To discuss the amplification of the second harmonic, the peak frequencies of the nonlinear parameter  $F_n$  are plotted for different normal stiffnesses in Fig. 4. It has been shown in the linear analysis<sup>4</sup> that an imperfect joint of plates subjected to the S0 mode incidence has two resonance frequencies depending selectively on the normal and tangential stiffnesses. In Fig. 4, the half of the resonance frequency depending on the normal stiffness  $F_T/2$  is shown together as a function of the normalized normal stiffness. The resonance frequency increases with increasing normal stiffness, and the peak frequency of the nonlinear parameter  $F_n$  shows in agreement with the half resonance frequency. For the incidence of the S0 mode at frequency  $f_0$ , the vibration of the double frequency  $2f_0$  induced at the imperfect joint due to the nonlinear interaction is symmetric with respect to the middle surface of the plate  $x_2 = 0$  because of the symmetric property of the incident S0 mode. When the double frequency  $2f_0$  corresponds to the resonance frequency of the imperfect joint, the resonance occurs and the generated S0 mode of frequency  $2f_0$  is amplified. Since the vibration at the double frequency  $2f_0$  is generated only in the normal direction of the interface, the nonlinear parameter shows only a single peak in Fig. 3.

## 4. CONCLUDING REMARKS

In this study, the second harmonic generation from an imperfect joint of elastic plates subjected to the Lamb wave incidence has been examined by perturbation analysis. The imperfect joint has been modeled as a nonlinear spring-type interface, which expresses the interfacial stress components as quadratic polynomials with respect to the interfacial displacement discontinuities. The displacement field has been expressed as the superposition of the fundamental wave and the second harmonic, and each wave field has been obtained in the frequency domain by the hybrid finite element method (HFEM). For the incidence of the lowest-order symmetric (S0) mode below the cut-off frequencies of the higher-order modes, it has been



**Figure 4.** Comparison of the peak frequency of the nonlinear parameter to the half resonance frequency of the imperfect joint. The above frequencies are given as normalized quantities.

shown that a nonlinear parameter calculated from the amplitude of the second harmonic S0 mode takes a sharp peak at a certain incident frequency. This peak frequency increases with increasing interfacial stiffness. The results of the linear analysis have indicated that the resonance behavior of the imperfect joint leads to the amplification of the second harmonic S0 mode.

## ACKNOWLEDGMENTS

This work has been supported by JSPS KAKENHI Grant numbers 26-2190, JP16H04235, and JP18K13663.

## REFERENCES

- <sup>1</sup> I. Y. Solodov, “Ultrasonics of non-linear contacts: propagation, reflection and NDE-applications,” *Ultrasonics* **36**, 383–390 (1998).
- <sup>2</sup> S. Biwa, S. Hiraiwa, and E. Matsumoto, “Experimental and theoretical study of harmonic generation at contacting interface,” *Ultrasonics* **44**, e1319–e1322 (2006).
- <sup>3</sup> N. Mori, S. Biwa, and T. Hayashi, “Reflection and transmission of Lamb waves at an imperfect joint of plates,” *J. Appl. Phys.* **113**, 074901 (2013).
- <sup>4</sup> N. Mori and S. Biwa, “Resonance of an imperfect joint of plates by the lowest-order symmetric Lamb mode,” *J. Acoust. Soc. Am.* **137**, 3139–3148 (2015).
- <sup>5</sup> N. Mori and S. Biwa, “Transmission of Lamb waves and resonance at an adhesive butt joint of plates,” *Ultrasonics* **72**, 80–88 (2016).
- <sup>6</sup> N. Mori and S. Biwa, “Transmission characteristics of the S0 and A0 Lamb waves at contacting edges of plates,” *Ultrasonics* **81**, 93–99 (2017).
- <sup>7</sup> Y. Shen and V. Giurgiutiu, “Predictive modeling of nonlinear wave propagation for structural health monitoring with piezoelectric wafer active sensors,” *J. Intell. Mater. Syst. Struct.* **25**, 506–520 (2013).
- <sup>8</sup> Y. Yang, C.-T. Ng, A. Kotousov, H. Sohn, H. J. Lim, “Second harmonic generation at fatigue cracks by low-frequency Lamb waves: Experimental and numerical studies,” *Mech. Syst. Signal Process* **99**, 760–773 (2018).
- <sup>9</sup> S. Biwa and N. Ito, “Nonlinear response of a closed defect to Lamb waves: perturbation analysis using hybrid finite element method,” *Recent Adv. Struct. Integr. Anal. – Proc. Int. Congr.*, Elsevier, pp. 261–264 (2014).